I. OBJECTIVE OF THE EXPERIMENT

Observation of the Hall Effect, and measurement of the Hall constant of a few semi-conductors and metals samples.

II. THEORETICAL BACKGROUND

When a current density $j = I/S$ flows through a sample (metal or semiconductor) that is immersed in a magnetic $\vec{B}$ field, perpendicular to the sample (Fig. 1), a Hall voltage $V_H \propto J_x \cdot B_z \cdot b$ is generated and can be measured along the sides of the sample. We define the proportionality constant $R_H$ such as $V_H = R_H J_x B_z b$. This phenomenon is called the Hall Effect and $R_H$ is the Hall constant.

Fig. 1  Electric conductor with cross-section $S = a \cdot b$ in a magnetic field $\vec{B}$.

The kinetic phenomena that occur in a sample due to the simultaneous effect of an electric field $\vec{E}$ and a magnetic field $\vec{B}$ are called galvanomagnetic. The Hall effect is one of the best known galvanomagnetic phenomenon.

Ohm’s law.

The vector equation of motion for a particle of charge $q$ and mass $m$ in a solid, undergoing an external force $\vec{F}_e$ and a friction $\vec{f} = -k\vec{v}$, is given by:

$$m \frac{d}{dt} \vec{v} = \vec{F}_e - \frac{m}{\tau} \vec{v}$$  \hspace{1cm} (1)
The friction force describes the relaxation of the particle due to its interactions (collisions) with the ions of the crystal lattice and with the other charge carriers. Here, we suppose that if the external forces go back to zero, the state returns to its equilibrium position exponentially with a relaxation time $\tau$. 

$$\frac{d\tilde{v}}{dt} = -\frac{1}{\tau} dt \quad \Rightarrow \quad \tilde{v} = \tilde{v}_0 e^{-t/\tau} \quad (2)$$

If the external forces remain constant, the system goes to a stationary state, i.e. $d\tilde{v}/dt = 0$. Supposing that the external force is due to a homogeneous electric field $\vec{E}$, the new stationary velocity, or drift velocity $\tilde{v}_d$, of the charge carrier becomes:

$$0 = q\vec{E} - \frac{m}{\tau} \tilde{v}_d \quad \Rightarrow \quad \tilde{v}_d = \frac{q\tau}{m} \vec{E} = \mu \vec{E} \quad (3)$$

The algebraic quantity $\mu = q\tau/m$ represents the speed per unit electric field, and is defined as the mobility of the charge carriers. For a solid containing $N$ charge carriers per unit volume, under the action of an electric field $\vec{E}$ and a permanent regime, then the charge carriers move with an average drift velocity in the same direction as the electric field. In this case, a constant electric current appears, given by:

$$\vec{j} = qN \tilde{v}_d = \frac{q^2N\tau}{m} \vec{E} = qN\mu \vec{E} = \sigma \vec{E} \quad (4)$$

where $\sigma = qN\mu$ is the electric conductivity. The equation $\vec{j} = \sigma \vec{E}$ is known as Ohm's law.

**Current density $\vec{j}$ in the presence of an electromagnetic field**

Assume a random $\vec{E}$ and $\vec{B} = B_z \hat{e}_z$ along the Oz axis. By defining the cyclotron frequency $\omega = (q/m)B_z$ and the electric conductivity $\sigma = qN\mu$ defined for $B = 0$, then it can be shown that in a stationary regime, the average drift velocity of the particle is given by:

$$\tilde{v}_d = \frac{q\tau}{m} \vec{E} + \frac{q\tau B_z}{m} (\tilde{v}_d \times \hat{e}_z) \quad (5)$$

and that the projection of the current density $\vec{j} = qN\tilde{v}_d$ on the x, y and z axes yields

$$j_x = \sigma \left( \frac{E_x + \omega \tau E_y}{1 + \omega^2 \tau^2} \right) \quad j_y = \sigma \left( \frac{E_y - \omega \tau E_x}{1 + \omega^2 \tau^2} \right) \quad j_z = \sigma E_z \quad (6)$$

In the presence of a magnetic field, the current density $\vec{j}$ is generally not parallel to the electric field $\vec{E}$. However for metals, even under a large applied magnetic field $B$, the corresponding anisotropy is very small, in such a way that $\vec{j} \approx \sigma \vec{E}$, i.e. Ohm’s law remains valid. The consequences of anisotropy are mostly relevant in semiconductors, and depend on the geometry of the system.
Hall Effect
Equation (6) shows that under a magnetic field \( \vec{B} = B_x \hat{e}_x \), the charge carriers are deflected towards the sides of the sample. On the other side, a lack of charges carriers creates an effective charge of opposite side. This charge separation continues until the voltage generated this way, called Hall Voltage, counters the magnetic force. At the equilibrium state, there is no longer a drift velocity along the Oy axis. Therefore the Hall field \( E_H \) is defined by the condition \( j_y = 0 \) and the equations in (6) allow us to find the Hall relation:

\[
E_H = E_y = \frac{\omega \tau}{\sigma} j_x = \left( \frac{1}{qN} \right) \left( j_x \cdot B_z \right) = R_H \cdot j_x \cdot B_z \tag{7}
\]

with \( R_H = \frac{1}{qN} \) \( \tag{8} \)

\( R_H \) is therefore an experimental measure of the algebraic quantity describing the mobile charge carrier density in a conductor, and the sign of the carrier. Also, if \( \sigma \) is known, then a measure of \( R_H \) can be used to determine the mobility \( \mu = R_H \sigma \), as long as there is only one type of carrier.

Different types of charge carriers
In semiconductors, the electric conductivity is often the result of two charge carriers of charges \( q_1 \) and \( q_2 \), of density \( N_1 \) and \( N_2 \), respectively. The total conductivity and the current can thus be written:

\[
\sigma = \sigma_1 + \sigma_2 = q_1 N_1 \mu_1 + q_2 N_2 \mu_2 \tag{9}
\]

\[
\vec{j}_H = \vec{j}_1 + \vec{j}_2 = q_1 \vec{v}_1 N_1 + q_2 \vec{v}_2 N_2 \tag{10}
\]

At room temperature, the relaxation time \( \tau \) is of the order \( 10^{-14}-10^{-15} \) s, so the term \( \omega \tau \) is of order \( 10^{-3} \) and the second order terms \( (\omega \tau)^2 \) are negligible. In this case, it can be shown that equation (6) can be written as:

\[
\begin{align*}
\vec{j}_x &= \sigma E_x + (\sigma_1 \mu_1 + \sigma_2 \mu_2) B_z E_y \\
\vec{j}_y &= \sigma E_y - (\sigma_1 \mu_1 + \sigma_2 \mu_2) B_z E_x
\end{align*}
\tag{11}
\]

and that the Hall condition \( j_y = 0 \) implies

\[
\begin{align*}
E_y &= \frac{\Sigma}{\sigma} E_x; \quad j_x = \left[ 1 + (\Sigma/\sigma)^2 \right] B_z E_x \equiv \sigma E_x \quad \text{with} \quad \Sigma = (\sigma_1 \mu_1 + \sigma_2 \mu_2) B_z, \quad \text{thus}
\end{align*}
\]

\[
R_H = \frac{(\sigma_1 \mu_1 + \sigma_2 \mu_2)}{(\sigma_1 + \sigma_2)^2} = \frac{q_1 N_1 \mu_1^2 + q_2 N_2 \mu_2^2}{q_1 N_1 \mu_1 + q_2 N_2 \mu_2} \tag{12}
\]

Example: two different charge carriers; electrons and holes with \( q_p = -q_e = q \)

\[
R_H = \frac{N_p \mu_p^2 + N_e \mu_e^2}{q(N_p \mu_p + N_e \mu_e)} \tag{13}
\]
The following table lists a few numerical values of the Hall coefficient $R_H$ of a few metals and semiconductors:

<table>
<thead>
<tr>
<th>Material</th>
<th>$R_H \left[10^{-10} \text{m}^3/\text{C}\right]$</th>
<th>Number of charge carriers $\left[10^{29} \text{m}^{-3}\right]$</th>
<th>Atomic density $\left[10^{29} \text{m}^{-3}\right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>-0.85</td>
<td>0.75</td>
<td>0.59</td>
</tr>
<tr>
<td>Au</td>
<td>-0.72</td>
<td>0.87</td>
<td>0.48</td>
</tr>
<tr>
<td>Bi</td>
<td>-5400.0</td>
<td>1.2 $10^{-4}$</td>
<td>0.28</td>
</tr>
<tr>
<td>Cu</td>
<td>-0.540</td>
<td>1.1</td>
<td>0.85</td>
</tr>
<tr>
<td>Fe</td>
<td>+0.228</td>
<td>0.06</td>
<td>0.84</td>
</tr>
<tr>
<td>Zn</td>
<td>+0.33</td>
<td>1.9</td>
<td>0.64</td>
</tr>
<tr>
<td>Si (doped with As) typically</td>
<td>At room temperature</td>
<td>$6 \times 10^7 \text{ à} 6 \times 10^4$ (*)</td>
<td>$10^{-8} \text{ à} 10^{-5}$</td>
</tr>
</tbody>
</table>

In metals, the sign of $R_H$ indicates whether the electric current is due to the motion of electrons ($R_H < 0$), or holes ($R_H > 0$). Holes are apparent positive charges, and could only be explained using ondulatory mechanics. (*) In semiconductors, the value and sign of $R_H$ is strongly dependent on the dopant density (“impurities”), $R_H$ can even be zero if $N_p\mu_p^2 = N_e\mu_e^2$.

III EXPERIMENTAL SETUP

Electromagnet power supply:

The setup is equipped with an electromagnet and a power supply according to the diagram in Fig. 2. The magnetic field $B$ in the gap of the electromagnet can be measured using a teslameter for calibration

*Ferromagnetism* (reminder, cf TP on hysteresis). When a current $I$ goes through a coil of length $L$, with $N$ turns, a homogenous magnetic field appears in the coil of magnitude $H = (N \cdot I)/L$ corresponding to the B-field $B_0 = \mu_0H$. If we put an iron core in the coil, the magnetic B-field increases greatly. An additional magnetic field $H'$ is added to the $H$ field, due to the magnetization of iron.

Magnetic induction is thus given by $B = \mu_0 (H+H')$. If we suppose a proportionality relation between $H$ and $H'$, we can rewrite $B = \mu_0 (H+\chi H) = \mu_0 (1+\chi) H = \mu_0H = \mu B_0$ with $\mu=(1+\chi)$ where $\mu$ is the relative magnetic permeability of the given material, and $\chi$ the magnetic susceptibility of the material. Ferromagnetic materials have a very high value for $\mu$ (up to $10^4$) but the $H'$ field is no longer proportional to $H$. For large values of $H$, $H'$ reaches an upper limit (saturation). On the other hand, is the magnitude of the $H$ field is reduced to zero, there is a remaining field $H'\neq 0$ (remnant). It is therefore necessary to apply an $H$ field of opposite direction (coercive field) to get rid of the remnant field (hysteresis phenomenon).

Hall voltage measurement

The Hall voltage $V_H = E_H \cdot b$ ($b =$ width of the sample) should be measured using a counter-voltage method. However, for simplicity, we will use a microvoltmeter with high entry impedance, that leads to very satisfying results (extra error generated is less than 0.1%). The diagram in figure 3 shows the setup of the used appliances.
Fig. 2  Experimental setup

Fig. 3  Diagram of the electric setup:
   a) semiconductor sample with 4 contacts
   b) plates with 5 contacts
IV SUGGESTED EXPERIMENTS

4-contacts samples (semiconductors)

For these measures, we will use strongly doped InP samples. The samples are prepared as thin layers (1-2 microns) and have a Hall cross, ideal for these types of measures (Fig. 3b). The symmetric configuration of the sample allows us to make 4 different measurements of $V_H$, for a fixed field $B^+$, by a simple cyclic permutation of the contacts 1234. If we invert the B field (to get $B^-$), we can take 4 more measurements. An image of the setup is shown in figure 4.

1) Measure the Hall voltage of Si doped InP (n-type). Setup the experiment like in fig. 3a, for instance with $I$ flowing from 1 to 3 and the Hall voltage from 2 to 4 ($I_{13}V_{24}$ configuration), and set the current to 1mA.
   - For $B=0$, measure the offset voltages $V_{ij}$ for the 4 possible configurations. Determine the origin of the residual voltage.
   - For a given configuration, measure the Hall voltage $V_H$ as a function of $B^+$.
   - Invert the field repeat the measurements of $V_H$ as a function of $B^-$. 

2) Set the magnetic field to a constant value (for example 400 mT, corresponding to something like 4 A in the coils) and measure $V_H$ as a function of the current in the sample. Vary current from -1 to 1 mA.

3) Calculate $R_H$ and the charge density $N$ for each case. Use linear fit from a graph.

4) How does the Hall voltage behave with respect to the sample thickness $a$?
## Sample Thickness $a$ Working current Maximal current

<table>
<thead>
<tr>
<th>Sample</th>
<th>Thickness $a$</th>
<th>Working current</th>
<th>Maximal current</th>
</tr>
</thead>
<tbody>
<tr>
<td>InP:Si</td>
<td>1 $\mu$m</td>
<td>1 mA</td>
<td>1 mA</td>
</tr>
<tr>
<td>InP:Si</td>
<td>2 $\mu$m</td>
<td>1 mA</td>
<td>1 mA</td>
</tr>
<tr>
<td>Ag</td>
<td>1.9 $\mu$m</td>
<td>2 A</td>
<td>3 A</td>
</tr>
<tr>
<td>Cu</td>
<td>1.6 $\mu$m</td>
<td>2 A</td>
<td>3 A</td>
</tr>
<tr>
<td>Bi</td>
<td>3 mm</td>
<td>2 A</td>
<td>3 A</td>
</tr>
<tr>
<td>W</td>
<td>0.6 $\mu$m</td>
<td>200 mA</td>
<td>300 mA</td>
</tr>
<tr>
<td>$\text{In}_2\text{O}_3:\text{SnO}_2$</td>
<td>0.15 $\mu$m</td>
<td>100 mA</td>
<td>150 mA</td>
</tr>
<tr>
<td>Bi</td>
<td>0.55 $\mu$m</td>
<td>30 mA</td>
<td>40 mA</td>
</tr>
</tbody>
</table>

**Tab1**: Thicknesses and currents allowed for each samples.

### 5-contacts samples (metals)

Samples with a 5-contacts configuration allow to get rid of the residual voltage measured with no $B$ field when a current flows in the sample. Therefore, we adjust the potentiometer (cf Fig 3b). This permits to measure in addition samples delivering greatly different values of Hall voltage with the same setup.

You have at your disposal six samples with this configuration. Please follow these guidelines:

1) Choose a sample. Connect it following the Fig. 3b). Setup the **working current** in the sample (cf Tab 1). With no $B$ field, adjust the potentiometer to get rid of the residual voltage. Determine how the system functions and the origin of this voltage.
   - (For all samples apart the W one) Put the working current through the sample and measure $V_H$ as a function of the magnetic field $B$. Follow the following path: $B \rightarrow 0 \rightarrow B^*$.  
   - Fix the $B$ field and measure $V_H$ as a function of the current in the sample. You can vary the current up to the maximal allowed value (cf Tab 1). **WARNING: Do not let the sample with the maximal current value more than the time to take the measurement, ie, some seconds!**

2) Choose another sample and repeat 1). Use as many as possible samples.

3) Determine $R_H$ and $N$ for each measurement. Use linear regression.

4) Conclude on the specificity of each materials

### Some remarks concerning the good procedings

- For this lab work, one has to identify vector objects with instruments only capable of measuring its norm. One should define at the beginning of the experiment (cf Fig.1) a Cartesian coordinates system and use it all the experiment along. For example one could call $B > 0$ the norm of the $B$-vector along $z$ and $-B < 0$ the norm of the same vector oriented in the opposite direction. Try to define the same quantities for the current density vector and the Hall electric field.

- **Do not exceed the maximal current allowed for each samples! Do not let any current flowing in the coils, neither in the sample, if you are not measuring anything!**

### Calibration of the B-field (bonus).

Using a Hall probe (teslameter) measure $B=f(|I_B|)$ for values of $0 \leq |I_B| \leq 6$ A. Measure the $B = f(I_B)$ following this specific path: $I_B^\text{Max} \rightarrow 0 \rightarrow -I_B^\text{Max} \rightarrow 0 \rightarrow I_B^\text{Max}$. Observe the effects of saturation and remnant field (hysteresis). Deduce the “useful” range of $B$. Knowing the calibration curve $f(I_B)$, what can you say about the uncertainty of $B$ at a specific known $I_B$?

### Using the Teslameter.

The teslameter is in a black box next to the setup. It must be zeroed before use. To do so, the probe should be isolated from any external magnetic field. To achieve this, place the tip of the probe in the metal cylinder. Then, set the button to Zero, and hold the button Hold Reset until the display shows 0. Then, put the button back on to Measure. While measuring a field, make sure the field is orthogonal to the probe. Please use the teslameter with caution. It’s a fragile measuring device.

### Bibliography:

Electric and magnetism classes.