I. OBJECTIVE OF THE EXPERIMENT

Can electric charge appear in any quantity, or only as an integer multiple of a fundamental unit, or quantum?

Many experiments have been thought up to answer this question. The most classical is that of the American physicist Robert A. Millikan (1869-1953), the oil drop experiment.

By observing the vertical motion of charged oil droplets in an electric field, he managed to find the value of the elementary charge, of which all electric charges in nature are multiples. This successful experiment was the reason for his Nobel Prize in 1923.

The objective of this experiment is to recreate Millikan’s experiment in order to show the quantification of charges.

II. INTRODUCTION: QUANTIFICATION OF ELECTRIC CHARGE

For his experiment, Millikan generated a vertical electric field between two parallel plates. This field could alternatively be on or off, and its direction could be switched. The upper plate has small perforations through which oil droplets could pass.

The observation of upward or downward motion of the oil droplets allowed Millikan to determine the value of the elementary charge.

\[ e = 1.6021 \times 10^{-19} \text{ C} \]

Two methods allow this value to be determined: either using a continuous electric field, or an alternating one. We will principally study the case of a uniform electric field. The description of the experiment for an alternating field can be found as an appendix.

Uniform continuous electric field

By inverting the uniform electric field back and forth, we can switch the direction of motion of the drop (up or down). We can then measure the time intervals \( \delta t_1 \) (respectively \( \delta t_2 \)) that the droplets require to do a certain distance \( \delta s \) while going down (respectively up).

The respective velocities \( v_1 \) and \( v_2 \) while going down or up are:

\[ v_1 = \frac{\delta s}{\delta t_1} \text{ et } v_2 = \frac{\delta s}{\delta t_2} \quad (1) \]
Calculations (see appendix I) show that the value of the particle’s charge is given by:

\[
q = \frac{9}{2} \pi \sqrt{\frac{\eta}{\rho g}} \frac{(v_1 + v_2) \sqrt{v_1 - v_2}}{U/d}
\]

where \( \eta \) is the air viscosity, \( d \) is the distance between the plates of the capacitor, \( \rho \) is the density of the oil, \( g \) is the acceleration of earth, and \( U \) is the voltage of the capacitor \((U/d = E)\).

### III. DESCRIPTION OF THE SETUP

The setup we shall use is pretty simple (Fig. 1). It’s made up of:

- A viewing unit, that allows for the droplets in the dielectric material to be observed (microscope, lighting and screen, see Fig. 2)
- A variable continuous voltage source \((0 \text{ V} - 250 \text{ V})\)
- A continuous voltmeter \((\text{set scale to } 0 \text{ V} - 500 \text{ V})\)
- A radioactive source (radium) that will be used to modify the charge of individual droplets. The drop actually catches certain ions of the air around it, and an X or \( \gamma \) ray source increases the air’s ionization.
- Two electric stopwatches \((\text{precision: } 1/100 \text{ s})\)

To charge a particle, hold the radioactive source behind the capacitor, facing it. An exposure time of a few seconds should be enough to charge several droplets. For safety purposes, make sure you always stow the radioactive source when not in use.

The mount has a switch that allows to change the direction of the electric field, or turn it off completely. This switch is coupled to the stopwatches, so that the stopwatches measure the precise time during which the electric field is applied.

In order to record data, you simply need to find a charged droplet and make it oscillate, between two lines of the screen using the switch, and write down corresponding travel times.

### Required Numerical Values for Calculations

Micrometer constant (for the used optical setup):
30 partitions, yielding: \((0.84 \pm 0.01) \cdot 10^{-3} \text{ m}\).

Distance between capacitor plates:
\[
d = (2.50 \pm 0.01) \cdot 10^{-3} \text{ m}.
\]

Density of the droplets:
\[
\rho = (898 \pm 1) \text{ kg/m}^3
\]

Air viscosity:

- at 0°: \( \eta_0 = (1.708 \pm 0.001) \cdot 10^{-5} \text{ Ns/m}^2 \)
- at 18°: \( \eta_{18} = (1.837 \pm 0.001) \cdot 10^{-5} \text{ Ns/m}^2 \)
- at 20°: \( \eta_{20} = (1.855 \pm 0.001) \cdot 10^{-5} \text{ Ns/m}^2 \)
- at 40°: \( \eta_{40} = (1.904 \pm 0.001) \cdot 10^{-5} \text{ Ns/m}^2 \)
Acceleration of gravity:

\[ g = (9.81 \pm 0.01) \text{ m/s}^2 \]

Fig. 1: Image of the setup for Millikan's experiment

Fig. 2: Schematic view of the experimental setup
IV  SUGGESTED EXPERIMENTS

1) Observe a droplet in a continuous field for a given value of the voltage $U_1$, calculate the speeds $v_1$ and $v_2$ basing your estimate on at least 3 measures in each case. Deduce the value of $q$ from this.

2) Select another droplets and repeat point 1) as many times as you can. The aim is to have around 30 to 50 droplet measured for the voltage $U_1$.

3) Select a new droplet and make 3 measurement of $q$ for a different voltage. Vary the voltage and repeat the measurement on the same droplet. Should the droplet charge depend on the voltage between the condenser plates?

Points to discuss:
- Compare all values of $q_i$ determined this way, and try to highlight integer multiples $n_i$ of the elementary charge ($e = 1.60 \times 10^{-19} \text{ C}$). Idea: Represent all the measured $q_i$ values in a histogram. Thus what would be the binning?
- Determine from your experiment your value of the fundamental charge.
- Discuss about the precision of $q_i$.
- Are there values of $n_i$ for which it isn’t possible to show charge quantification $q_i$?

Optional:
- Do the experiment with the alternating electric field instead of the continuous one (see appendix 2).
- Visually estimate the diameter of a droplet, and compare it to the value obtained from equation (11). What do you think?

Comments:
1) Sometimes, the droplet leaves the vertical plane where it is supposed to go up and down. Its sharpness decreases. This can be corrected by adjusting the focus of the microscope

2) Thermal agitation of the air molecules in the capacitor do slightly interact with the system, but shouldn’t have too much of a negative impact on the numerical results

V. BIBLIOGRAPHY


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3) G. Bruhat : "Cours de Physique Générale : Electricité" Masson & Cie (p. 737 et suivantes)

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VI. APPENDIX

The application of a vertical electric field (uniform or alternating) in the capacitor sets the oils droplets in the dielectric material (air here) in motion.

The following forces apply to the droplet:

- $\vec{F}_g$: gravitational force
- $\vec{F}_E$: electrical force
- $\vec{F}_R$: resistance of the dielectric medium to the motion of the droplet
- $\vec{F}_p$: Archimedes force (negligible with respect to $\vec{F}_R$)

and can be expressed by the following parameters:

- $\vec{v}$: droplet’s velocity
- $U$: capacitor’s voltage
- $d$: distance between the plates of the capacitor
- $q$: droplet’s charge
- $r$: droplet’s radius (supposed spherical)
- $\rho$: oil density
- $g$: gravitational acceleration
- $\eta$: viscosity of the dielectric medium

We get:

\[
\vec{F}_g = mg = \frac{4}{3} \pi r^3 \rho \bar{g}
\]

\[
\vec{F}_g = q\vec{E} = -q\vec{E} U
\]

\[
\vec{F}_g = -6\pi \eta r \bar{v}
\]

(see experiment B2)

It is therefore possible to calculate the value of the charge $q$ of the observed droplet.

The observation of the studied motion allows us to suppose the motion is uniform (vertical, if the capacitor’s plates are horizontal).

We can represent the forces applied to a droplet on Fig. 3.

![Fig. 3: Forces applied on a droplet](image_url)
Uniform electric field

According to Newton’s law, \( \vec{F} = m\vec{a} = m \frac{dv}{dt} = 0 \)

Thus

going down: \[ qE_1 + mg - F_{R_1} = 0 \] (5)

going up: \[ qE_2 - mg - F_{R_2} = 0 \] (6)

With
\[ E_1 = E_2 = U/d \]
\[ F_{R_1} = 6\pi\eta r_1 \]
\[ F_{R_2} = 6\pi\eta r_2 \]

Equations (5) and (6) respectively become:
\[ q \frac{U}{d} + \frac{4}{2} \pi r^3 \rho g - 6\pi\eta r_1 = 0 \] (7)
\[ q \frac{U}{d} + \frac{4}{2} \pi r^3 \rho g - 6\pi\eta r_2 = 0 \] (8)

Combining (7) and (8) in two different ways yields:

(7)+(8) : \[ r = \frac{q U/d}{3\pi\eta(v_1 + v_2)} \] (9)

(7)-(8) : \[ r = \frac{3}{2} \sqrt{\frac{\eta(v_1 - v_2)}{\rho g}} \] (10)

Comparing (9) and (10):

\[ q = \frac{9}{2} \pi \sqrt{\frac{\eta^3 d^2 (v_1 + v_2) \sqrt{v_1 - v_2}}{\rho g U}} \] (11)

Finally, by using the distance \( \delta s \) travelled by the droplets during times \( \delta t_1 \) and \( \delta t_2 \), we get

\[ q = \frac{9}{2} \pi \sqrt{\frac{\eta^3 d^2 \left( \frac{1}{\delta t_1} + \frac{1}{\delta t_2} \right)}{\rho g U}} \left( \delta s \right)^{3/2} \] (12)
**Alternating electric field**

We apply a sinusoidal voltage

\[
U(t) = U_0 \sin \omega t
\]  

(3)

to the capacitor. The droplets oscillate vertically at the same frequency, \(U_0\) being the peak value.

Calculations lead to the following charge \(q\) (see below):

\[
q = \frac{q}{2\pi \omega} \sqrt{\frac{\eta^3}{\rho g \frac{U_{eff}}{d}}} \sqrt{\nu_c}
\]

(4)

where we add the following definitions to those above:

- \(A\) amplitude of the droplet’s motion
- \(U_{eff}\) effective applied voltage
- \(\nu_c\) velocity of the droplet in free fall

Let \(v\) be the velocity of the oscillating droplet in the alternating sinusoidal field, we have:

\[
U(t) = U_0 \sin \omega t, \ U_0 \text{ being peak value of } U(t)
\]

\[
E(t) = \frac{1}{d} U_0 \sin \omega t
\]

The motion of the droplet is a combination of the oscillation of frequency \(\omega\) and of its free fall motion that can be noticed after a time \(\Delta t\) (see Fig. 4)

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**Fig. 4**: Loss of height of an oscillating droplet

The observed amplitude \(|Z|\) is practically independent of the free fall motion, in the calculations that follow. The only acting forces are:

\[
F_R = 6\pi \eta r \cdot v(t)
\]

\[
F_E = \frac{q}{d} U_0 \sin \omega t
\]
According to Newton’s law:

\[
m \cdot \ddot{z} = \frac{q \cdot U_0 \cdot \sin \omega t}{d} - 6 \pi \cdot \eta \cdot r \cdot \ddot{z}
\]  \hspace{1cm} (13)

But:

\[
m \cdot \ddot{z} + 6 \pi \cdot \eta \cdot r \cdot \ddot{z} = \frac{q \cdot U_0}{d} \cdot \sin \omega t
\]  \hspace{1cm} (14)

The solution to this differential equation in the stationary case allows us to calculate the amplitude \( |z_0| \) of the drop’s motion.

We look for a solution of this type: \( z = z_0 \cdot \sin(\omega t + \varphi) \) and get:

\[
-m \cdot z_0 \cdot \omega^2 \cdot \sin(\omega t + \varphi) + z_0 \cdot \omega \cdot 6 \pi \cdot \eta \cdot r \cdot \sin(\omega t + \varphi + \frac{\pi}{2}) = \frac{q \cdot U_0}{d} \cdot \sin \omega t
\]

thus:

\[
|z_0| = \frac{q \cdot U_0}{\omega d \cdot \sqrt{m^2 \cdot \omega^2 + 36 \cdot \pi^2 \cdot \eta^2 \cdot r^2}}
\]  \hspace{1cm} (15)

By accounting for the numerical values obtained during the experiment, we can show that \( m^2 \cdot \omega^2 << 36 \cdot \pi^2 \cdot \eta^2 \cdot r^2 \) and we can that approximate equation (15):

\[
|z_0| = \frac{q \cdot U_0}{6 \pi \cdot \eta \cdot r \cdot d \cdot \omega}
\]  \hspace{1cm} (15*)

The measure of this double amplitude \( A = 2|z_0| \) is easier to determine, and if we consider the effective voltage \( U_{\text{eff}} = U_0 / \sqrt{2} \) we get:

\[
q = \frac{3 \cdot \pi \cdot \eta \cdot r \cdot d \cdot \omega}{2 \cdot \sqrt{2} \cdot U_0} \cdot A
\]  \hspace{1cm} (16)

By observing the same droplet in free fall (no electrical field) lets us evaluate its radius \( r \). In free fall, since the motion is uniform,

\[
v_c \cdot \vec{F}_R + m \vec{g} = 0
\]

so:

\[
6 \pi \eta r v_c = \frac{4}{3} \pi r^3 \rho
\]  \hspace{1cm} (Newton’s law).

and finally

\[
r = \sqrt{\frac{9 \cdot \rho \cdot v_c}{2 \cdot \eta \cdot g}}
\]  \hspace{1cm} (17)