I. OBJECTIVE OF THE EXPERIMENT.

Swissmetro travels at high speeds through a tunnel at low pressure. It will therefore undergo friction that can be due to:

1) Viscosity of gas (cf. "Viscosity of gas" experiment)  
2) The air in front of the object being pushed out of the way ("piston effect")  
3) Drag forces

Through calculation only, it is difficult to estimate the total resistance forces that are applied to the object, since they depend on numerous geometrical parameters (aerodynamics of the object, distance separating object from wall, ...) but also physical parameters (gas pressure, velocity, ...)

The following experiment is a model that will give a global idea of the resistance force acting on the object in motion, and determine how these forces decrease with air pressure.

II. MOTION RESISTANCE

Newton’s equation applied to a fluid yields Bernoulli’s equation:

\[ P + \rho \frac{v^2}{2} + \rho gh = \text{const} \quad (1) \]

where \( P \) is pressure, \( \rho \) is density, \( v \) is speed, \( h \) is height and \( g \) is gravitational acceleration.

Also, an object travelling at a speed \( v \), in a motionless fluid of pressure \( P_0 \) and density \( \rho_0 \), creates an impact point on the fluid in the area where the pressure is \( P_{\text{imp}} \), and the density \( \rho_i \) for a constant height, equation (1) yields:

\[ P_{\text{imp}} = P_0 + \rho \frac{v^2}{2} \quad (2) \]

The resistance force opposing to the object’s motion in a tunnel generally depends on the geometry of the vehicle. The pressure at the impact point, expressed in (2), should reduce the more we move away from that point.
We can express the force at the front of the vehicle by:

\[ F_1 = S \cdot P_{imp} = S(P_0 + K \rho \frac{v^2}{2}) \]  

(3)

where \( S \) is the cross section of the object and \( K \) a constant.

On the back of the vehicle, the pressure is equal to the average pressure acting on the wake, and can be expressed as:

\[ F_2 = S(P_0 + K' \rho \frac{v^2}{2}) \]  

(4)

In these conditions, the total resistance force acting on the vehicle behaves like:

\[ F = F_1 - F_2 = S(K + K')\rho \frac{v^2}{2} \]  

(5)

This force therefore depends on the cross section of the vehicle, its geometry \( (C_x) \), its speed, and the gas density \( \rho \). However, density depends on pressure. At zero pressure, there is no more gas, and therefore the force must vanish.

For an adiabatic compression: \( PV^\gamma = \text{const} \)  

(6)

And after differentiation:

\[ \frac{dP}{P} = -\gamma \frac{dV}{V} = \gamma \frac{d\rho}{\rho} \]  

(7)

since \( \frac{d\rho}{\rho} = -\frac{dV}{V} \)

and thus, from (7):

\[ \rho = \gamma P \frac{d\rho}{dP} = \kappa \cdot \gamma \cdot P \quad \text{with} \quad \kappa \frac{d\rho}{dP} = \text{const} \]  

(8)

and finally:

\[ F = SC_x \cdot \gamma \cdot P \cdot \frac{v^2}{2} \quad \text{with} \quad C_x = \kappa(K + K') \leq 2 \]  

(9)

**Viscous forces**

The Reynolds number \( Re \) determines the limit between laminar and turbulent flow.

\[ Re = \frac{\rho v D}{\mu} \]  

(10)

where \( v \) is fluid speed, \( D \) is distance between two walls where the fluid flows and \( \mu \) is dynamic viscosity coefficient.
In general, when the Reynolds number reaches approximately 2'200, the flow becomes turbulent, i.e. the velocities become random. Please note that Re decreases with pressure and distance between the walls. In a vacuum of 665 Pa and a distance of 4 cm, Re is equal to 2'400. We could therefore suppose a laminar flow between Swissmetro and the walls, as long as \( P \approx 665 \) Pa and the train-wall distance is inferior to 4 cm.

If the train-wall distance is of the order of a few centimeters, the flow is laminar (see below). In these conditions, the friction is given by:

\[
F_{\text{vis}} = \mu \frac{\Sigma}{d} v
\]  

(11)

with \( v \) is vehicle speed, \( \Sigma \) is cross section of the vehicle, \( d \) is train-wall distance, \( \mu \) is dynamic viscosity coefficient. The viscosity of a gas is relatively independent of the pressure down to 100 Pa and depends on the type of gas, and the temperature.

For air at 20°C \( \mu = 1.84 \cdot 10^{-5} \) Pa·s.

We can verify that the viscous friction is negligible with respect to the other resistance forces.

This “extra” vacuum is due to a poor \( C_s \) of the train travelling through a tube with a small amount of space between the walls.

### III. EXPERIMENTAL SETUP

The experimental setup allows us to study the global forces acting on the object when the tube is filled at different pressures.

In order to simulate a tube of infinite length, we will use a looped tube (Fig. 1 and Fig. 2). We can also limit the length of the tube (piston effect) by closing a valve on the circuit.

In one of the tube’s branches, there is a weight of mass \( m \) that can fall freely in the tube. The equation of motion is:

\[
mg - F_j = m\ddot{x}
\]

where \( F_j \) is the friction of the air on the weight (we will neglect the friction of the walls of the tube).

If we can measure the acceleration \( \ddot{x} \) as a function of speed, \( \ddot{x} = \ddot{x}(v) \), we can determine the function \( F_j = F_j(v) \) and decide which of the expressions (9) or (11) is more suitable.

In the event of a force like in (9), we can try to find the mobile’s \( C_s \) coefficient.

**Examples:**
- Disk moving perpendicularly to its surface \( C_s \approx -1.3 \)
- Sphere \( C_s \approx -0.5 \)
- Half sphere and cone (rain drop) \( C_s \approx -0.04 \)
With the available setup, we can determine five different speeds along the path:

pos. 0 \( t = 0 \) \( v = 0 \)
pos. 1 \( t = t_1 \) \( v = v_1 \)
pos. 2 \( t = t_2 \) \( v = v_2 \)
pos. 3 \( t = t_3 \) \( v = v_3 \)
pos. 4 \( t = t_4 \) \( v = v_4 \)

The speeds are determined from the dwell time of the mobile through an optical gate. It is therefore necessary to know the length of the weight.

From these values, we can determine 4 mean acceleration values for “\( a \)”:

\[
a_i = \frac{v_i}{t_i}
\]

\[
a_s = \frac{v_2 - v_1}{t_2 - t_1}
\]

etc.

These values give us an idea of \( F_j = F_j(v) \); for instance, we can verify whether \( F_j = \eta \cdot v \) or \( F_j = \eta \cdot v^2 \) supposing \( \eta \) constant.

Supposing the equation is of the type \( F_j = \eta \cdot v \), we can solve the equation below

\[
\ddot{x} + (\rho / m) \dot{x} = g
\]

**Solution:**

\[
\dot{x} = v = \frac{mg}{\eta} \left(1 - e^{-\frac{t}{\tau}}\right)
\]

where \( \tau = \frac{m}{\eta} \) is a relaxation time.
V. SUGGESTED EXPERIMENTS:

1) Measure the speeds of the mobile \( m \) as a function of the pressure in the tube, for both the finite and infinite tube.

2) Calculate the acceleration and determine the variation of friction:
   1 - with pressure
   2 – with speed.

3) Under what conditions is \( F_j = \eta \cdot v \) valid (laminar flow)?

4) Discuss the results of the finite and infinite tube according to its application for Swissmetro. (Can Swissmetro push the air in front of it as if it were and infinite tube?)

Operating instructions

Turn on the main power, and measure the system at atmospheric pressure, before starting the pumping process. Make sure the leak valve is closed, and open both valves before starting the pump. Regularly close valve 2, and perform a measure. Before continuing the pumping process, make sure that valve 1 is open. Take 20 to 30 measurements between atmospheric pressure and approximately 1 Torr.

Performing a measurement: In order to take a measurement, lift up the weight using the coil around the tube, and place it on the wedge towards the top of the tube. The current flowing through the coil can be adjusted using the current source (make sure the current is not too high). Then, reset the stopwatch. Press and hold the \textit{Measure} button to let go of the weight, and start the stopwatch. Repeat the measurement by switching the position of the valve 1. The pressure is first measured using a bourdon gauge (in mbar) and then by a Pirani gauge (in torr).

Useful quantities:

Mass of object: 10.7 g
Length of object: 76 mm
Diameter of object: 22.4 mm
Internal diameter of tube: 24.2 mm

Fig 2: Experimental setup